Set Union

The union of two sets is a new set formed by combining the membership of the original two sets. Thus, if we have $A=\{3,4,7,9\}$ and $B=\{1,3,5,7,9)$ we can form the union of the two sets, written as $A \cup B$ as the set $\{1,3,4,5,7,9\}$. As an equation we would say as $A \cup B=\{1,3,4,5,7,9\}$. As before, we do not repeat elements of a set. Just because $3 \in A$ and $3 \in B$ does not mean that we have two 3's in the union.

Let us continue by looking at another set, $D=\{5,7\}$. Then $D \cup A=\{3,4,5,7,9\}$ and $D \cup B=\{1,3,5,7,9\}$. You might note that in this last example the answer is the same as B. That is $D \cup B=B$. This is true because $D \subseteq B$.

Consider $E=\{2,4,6,8\}$. Then, $E \cup B=\{1,2,3,4,5,6,7,8,9\}$. And, we might note that $B \cup E=\{1,2,3,4,5,6,7,8,9\}$. The fact that $B \cup E=E \cup B$ is not unique to the sets B and E. By the very definition of the union of two sets, the order in which we give the sets makes no difference in the result. Taking the union of two sets produces a new set just like taking the sum of two numbers produces a new number. In that sense, union is an "operator" just as addition is an operator. When the order of the things the operator is working on makes no difference, as in $3+4=4+3$ or $B \cup E=E \cup B$, we say that the operation is commutative. Union is a commutative operator.

Adding a fifth set to our list, $F=\{1,4,7,11\}$ we might find $F \cup B \cup A$ as an expression. We need to decide how to do this and there are two choices: we could do $F \cup B$ first and take that answer union A, which we would write as $(F \cup B) \cup A$, or we could take F union the result of $B \cup A$, which we would write as $F \cup (B \cup A)$.

Again, from the definition of the union, we will get the same answer either way. This, again, is not unique to our sets F, B, and A. This will be true for any sets P, Q, and R. It is always the case that $P \cup R \cup Q = (P \cup R) \cup Q = P \cup (R \cup Q)$. An operator that has this property is called associative. Union is an associative operator.

Then too, we should observe that $\varnothing \cup E = E \cup \varnothing = E$, and, again, this is not unique to our set E. Because taking the union of the empty set with any set produces that other set as the answer we call the empty set the identity element for the operation of taking the union of two sets.